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An Approximate Method of Estimating Shear Velocity from Specific Heat[1]

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The purpose of this letter is to point out that the isotropic shear velocity v_s of inorganic materials such as of minerals and rocks can be estimated from low-temperature specific heat measurements. This property promises to be very useful because it enables one to determine the seismic shear velocity of a material without regard to the state of aggregation of the samples. This correspondence between acoustics and calorimetry is based upon a principle of lattice dynamics, namely, that at sufficiently low temperatures the optical vibrations of a solid are quiescent and the vibrational energy results solely from acoustic vibrations. This correspondence is conveniently stated in terms of Debye temperatures. It should be remarked that our procedure is essentially the reverse of the argument used by *Birch* [1952], who estimated the Debye temperature of the mantle from the seismic *S* velocity.

The low-temperature specific heat is represented by a scalar parameter called the thermal Debye temperature θ_t, and the acoustic contribution of specific heat is represented by the acoustic Debye temperature θ_a. Thus at temperatures near absolute zero

$$\theta_t = \theta_a \tag{1}$$

The expression for θ_a in terms of the sound velocities for an isotropic body is given by [*Barron*, 1957; *Grüneisen*, 1926].

$$\theta_a = \frac{h}{k}\left(\frac{9\rho N}{4\pi M/p}\right)^{1/3}\left\{\frac{2}{v_s^3}+\frac{1}{v_p^3}\right\}^{-1/3} \tag{2}$$

where h, k, and N are physical constants, M/p is the mean atomic weight (molecular weight divided by the number of atoms p determining the molecular weight), ρ is the density, and v_s and v_p are the shear and longitudinal velocities.

It is more convenient to write (2) in terms of the mean sound velocity v_m, or

$$\frac{3}{v_m^3} = \frac{2}{v_s^3} + \frac{1}{v_p^3} \tag{3}$$

Combining the foregoing equations, we have

$$\theta_t = 231.3\,[\rho p/M]^{1/3} v_m \tag{4}$$

Here the units of v_m are km/sec, and the numerical factor is the result of the physical and numerical constants in (2). We observe that both v_p and v_s are needed to define θ_t. A closer examination, however, reveals that to a good approximation v_s alone defines θ_t. This can be demonstrated by solving for the ratio of v_s/v_m in (3) and substituting for v_p/v_s the equivalent function of Poisson's ratio σ.

$$\left(\frac{v_p}{v_s}\right)^2 = 1 + \frac{1}{1-2\sigma}$$

$$\frac{v_s}{v_m} = \left[\frac{2}{3} + \frac{1}{3}\left(1+\frac{1}{1-2\sigma}\right)^{-3/2}\right]^{1/3} \tag{5}$$

Equation 5 is a slowly varying function of σ which is plotted in Figure 1. We see that v_s/v_m only varies $\pm 1.1\%$ about the value 0.9, for changes of σ of ± 0.1 about the value $\sigma = 0.25$. The Poisson ratio of the majority of materials lies between 0.15 and 0.35, so that within an approximation of a few per cent we have $v_s = 0.9 v_m$ and (4) is replaced by

$$v_s \approx (\theta_t/280)(M/p\rho)^{1/3} \text{ km/sec} \tag{6}$$

Equation 6 is the desired equation which has practical uses in geophysics. v_s can be computed if the mean atomic weight, density, and the low-temperature specific heat (from which θ_t

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